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A TECHNIQUE OF OBJECTIVE ANALYSIS AND INITIALIZATION FOR THE PRIMITIVE FORECAST EQUATIONS

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ABSTRACT

A technique of initialization for the primitive forecast equations is presented. The method consists of a marching prediction scheme performed in such a manner that the large-scale solution remains approximately steady, and high-frequency modes created in the adjustment process are damped selectively by time differencing with the Euler-backward method. The scheme places no restriction on the wind divergence field and ensures truncation error consistency between initialization and forecast equations.

1. INTRODUCTION

The mutual adjustment process between mass and velocity fields under the influence of the rotation and gravity of the earth is one of the most interesting physical characteristics of motion in the atmosphere and the oceans (Rossby, 1937–1938*a*, 1937–1938*b*; Cahn, 1945; Bolin, 1953; Obukhov, 1949; and Phillips, 1963). This process disperses the energy of the deviations from the equilibrium state between the pressure and Coriolis forces,[3] and, in the case of the atmosphere, maintains the quasi-geostrophic state of the large-scale motion that is observed on weather charts. In addition, it seems to the present authors that dissipative forces in the free atmosphere act rapidly to suppress the high-frequency sound-gravity-inertia motions and thus contribute to the balanced state. The lower frequency modes of these motions are not as responsive to viscous forces as those of high frequency and may have a life of several hours (e.g., tidal motions).

It is well known that observational data in the atmosphere contain excessive imbalances between the Coriolis and pressure forces and that these deviations lead to large-amplitude gravity-inertia waves if time integrations are attempted with the primitive equations (Richardson, 1922). The sources of these imbalances may lie in analyses as much as in observational errors. Although actual imbalance does occur in the atmosphere, it is doubtful that much of this is accurately revealed in the data. Perhaps areas of imbalance are implied, e.g., those of cyclostrophic change, and frontal and convective activity, but accurate representation of the excess is impossible.

A general balanced initialization may be attempted with diagnostic forms of the second-order filtered equations. The theory behind this approach has developed gradually since Hinkelmann (1951) proposed geostrophic balancing. The lack of flow curvature influences in the geostrophic method was improved in the balance approach suggested by Charney (1955); but the use of the "balance equation" gave only rotational characteristics of the large-scale flow. It was pointed out by Hinkelmann (1959) and Phillips (1960) that a lack of a small divergence in the initial data also causes excitation of high-frequency modes and that a good estimate of the divergent characteristics could be obtained through use of the quasi-geostrophic ω-equation. Later, several authors (Hinkelmann, 1961; Miyakoda, 1963; and Krishnamurti and Baumhefner, 1966) included the terms of second-order approximation in the filtered equations and utilized approaches involving simultaneous solution of the balance and ω-equations. All the above methods have had mathematical limitations concerning the existence and uniqueness of solution under a given boundary condition, i.e., the so-called ellipticity condition, but Fjørtoft (1962) has proposed a method of solution applicable in elliptic as well as hyperbolic areas.

The mathematical and physical approaches taken in initializations with the filtered equations give, in all cases, inexact conditions of balance when applied to the primitive equations. The more refined second-order equations may approach the balanced state supposed by the primitive equations, but a perfect agreement on the state of mutual adjustment can never be attained. When one adds to this problem the complexities introduced when secondary physical effects are considered (e.g., flow over mountains, radiation, friction, sensible heating, convection, etc.), the task of maintaining balance consistent with the primitive forecast equations becomes even more difficult.

The first published departure from the early conventional approach of initialization was taken by Miyakoda and Moyer (1968). The design of their scheme makes it more applicable for use with the primitive equations and, in particular, well suited for the addition of heating, friction, and convection simulations within the dynamic framework.

The scheme avoids the problem of the ellipticity condition present in previous approaches but retains a restriction on the horizontal divergence (the first and second individual time derivatives of divergence are assumed to be zero) normally employed in conventional initializations. Admittedly, this approximation is quite good on the synoptic scale at middle latitudes. The application to other situations is, however, quite limited.

Finally, the Miyakoda-Moyer scheme cannot ensure a truncation error consistency between the initialization equations and the forecast equations. The unique char-

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[3] Balance of forces in the atmosphere on the earth should (strictly speaking) include consideration of cyclostrophic effects as well as the effect of large-scale divergence.

acter of the numerical initialization produces truncation inconsistencies that can lead to imbalances in the interior of the grid and particularly at the boundaries when the forecast is begun. Any imbalance due to truncation differences can be expected to lead to fictitious gravity waves when the forecast is begun.

2. INITIALIZATION BASED ON THE PRIMITIVE FORECAST EQUATIONS

The initialization procedure presented here utilizes the primitive forecast equations directly. It departs from the Miyakoda-Moyer scheme in two ways. First, no explicit restriction on the horizontal divergence is required. Second, although the same time-differencing scheme (the "Euler-backward" method) is employed, it is applied in a different manner; i.e., an actual iteration of forward and backward forecasts are performed around the initial time. The method corresponds to a normal, marching prediction scheme where the large-scale field is quasi-steady, and only the high-frequency modes change with time.

The possible advantages to be gained with this approach are 1) it eliminates the truncation inconsistency problem discussed above and 2) it may have application in a wider range of problems since no restrictions are introduced in the motion fields.

It is the opinion of the present authors that the above departures could prove important under many circumstances. The adjustment properties of any particular primitive equation model are unique, not completely related to those of the atmosphere or any other model. This can be realized simply by recognizing the essentially linear nature of the adjustment process. (The nonlinear terms in the equations of motion influence the adjustment properties, but the pressure-gradient force and the Coriolis force dominate the process. Recently, Blumen, 1967, discussed the nonlinear effect in the geostrophic adjustment process.) The boundary conditions and error due to discrete representation of the continuous fields may excite gravity-inertia waves unique to the particular model. And afterward, the dispersive effects would dissipate the resultant noise in a manner peculiar to the mathematical formulation. It would seem that, under these conditions, mathematical consistency down to truncation equivalence should be considered in the design of initialization forecast schemes.

In the approach to a balanced state in the present scheme, data errors, numerical errors, etc. can be expected to lead to fictitious noise just as with the forecast equations. Since these excitations are at the source computationally (as opposed to physically) generated, it does not seem inconsistent to remove this noise by a computational artifact. A time-integration scheme that accomplishes this and, to some extent, simulates the viscous effect of damping high-frequency oscillations is the Euler-backward method or its modified version (Matsuno, 1966*a*, 1966*b*; Kurihara, 1965).[4] The damping behavior of the scheme has been shown to depend on the time step, as well as the frequency of oscillations.

A simple form of a prognostic equation,

$$\partial u/\partial t=F(u), \tag{1}$$

in the Euler-backward time difference may be expressed as follows:

$$u^*=u^\tau+F(u^\tau)\Delta t$$

and

$$u^{\tau+1}=u^\tau+F(u^*)\Delta t. \tag{2}$$

The symbol Δt represents the time increment over which the extrapolation is made. If a completely reversible differential equation is treated and integrated forward and then backward in time, the original solution will be obtained. However, if the Euler-backward method is employed with a finite Δt, a damping effect is introduced on the high-frequency physical modes. After one forward-backward forecast cycle, a wave of angular frequency ω experiences a reduction rate in amplitude given by

$$(1+i\omega\Delta t-\omega^2\Delta t^2)(1-i\omega\Delta t-\omega^2\Delta t^2)=1-\omega^2\Delta t^2+\omega^4\Delta t^4 \tag{3}$$

where $i=\sqrt{-1}$. For waves where $\omega\Delta t<1$, amplitude is reduced.

Thus, with initial estimates of the wind and mass fields an iteration is performed, and the original wind or mass field (whichever is most accurate) is restored after each cycle (see fig. 1). With a properly chosen time step, the high-frequency waves generated by errors and unique model characteristics are gradually reduced in amplitude; and the process is continued until a slowly changing initial state is achieved, void of high-frequency oscillations. In this respect, the criterion for convergence is somewhat arbitrary. Furthermore, it is not assured that along with the elimination of noise in the calculation a meteorologically significant balance may be realized.

These uncertainties will be investigated in two ways. First, an analysis of a simple linear system will be performed relating to a problem of barotropic, frictionless flow. Secondly, a nonlinear experiment is presented based on numerical simulations of a simple two-layer model.

3. INITIALIZATION OF A SIMPLE LINEARIZED SYSTEM

We shall consider the linearized equations of motion which govern an incompressible homogeneous atmosphere over the rotating flat domain,

$$(\partial V/\partial t)+fk\times V=-\nabla\phi \tag{4}$$

and

$$\partial\phi/\partial t=-H\nabla\cdot\mathbf{V}. \tag{5}$$

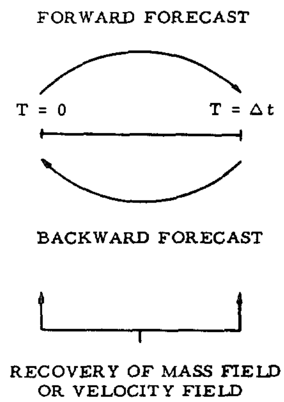
We transform equations (4) and (5) into a forward difference form, i.e.,

$$\mathbf{V}^{(\tau+1)}-\mathbf{V}^{(\tau)}=-\beta k\times\mathbf{V}^{(\tau)}-\gamma\nabla\phi^{(\tau)} \tag{6}$$

and

$$\phi^{(\tau+1)}-\phi^{(\tau)}=-\gamma H\nabla\cdot\mathbf{V}^{(\tau)} \tag{7}$$

[4] Through personal communication one of the authors has found that Matsuno (1967) and Gambo and Okamura (1967) have also experimented with this scheme in initialization and forecast problems in a manner similar to that discussed here.



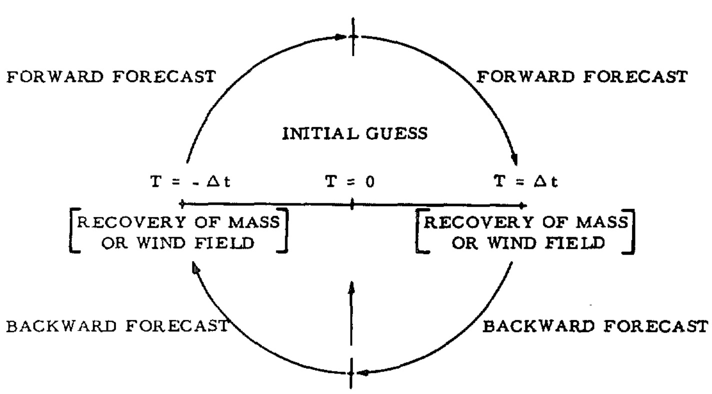


FIGURE 1.—Schematic representation of iteration methods for initialization with the primitive forecast equations.

where $\beta=f\Delta t$, $\gamma=\Delta t/(2\Delta s)$, and $\nabla=(\nabla_x, \nabla_y)=\{(\ )_{i+1,j}-(\ )_{i-1,j},\ (\ )_{i,j+1}-(\ )_{i,j-1}\}$. If we denote $A=u^{(\tau)}$, $B=\mathbf{V}^{(\tau)}$, $C=u^{(\tau+1)}$, $D=\mathbf{V}^{(\tau+1)}$, $G=\phi^{(\tau+1)}$, and $\phi=\phi^{(\tau)}$, equations (6) and (7) become

$$C^{(\nu)}=A^{(\nu)}+\beta B^{(\nu)}-\gamma\nabla_x\phi, \tag{8}$$

$$D^{(\nu)}=B^{(\nu)}-\beta A^{(\nu)}-\gamma\nabla_y\phi, \tag{9}$$

and

$$G^{(\nu)}=\phi-\gamma H(\nabla_x A^{(\nu)}+\nabla_y B^{(\nu)}), \tag{10}$$

where a superscript ν refers to the νth iteration.

The application of the Euler-backward system for the forward forecast results in the following:

$$C^{*}=A^{(\nu)}+\beta B^{(\nu)}-\gamma\nabla_x\phi, \tag{11}$$

$$D^{*}=B^{(\nu)}-\beta A^{(\nu)}-\gamma\nabla_y\phi, \tag{12}$$

$$G^{*}=\phi-\gamma H(\nabla_x A^{(\nu)}+\nabla_y B^{(\nu)}), \tag{13}$$

$$C^{(\nu)}=A^{(\nu)}+\beta D^{*}-\gamma\nabla_x G^{*}, \tag{14}$$

$$D^{(\nu)}=B^{(\nu)}-\beta C^{*}-\gamma\nabla_y G^{*}, \tag{15}$$

and

$$G^{(\nu)}=\phi-\gamma H(\nabla_x C^{*}+\nabla_y D^{*}). \tag{16}$$

Next, we perform a backward forecast with the Euler-backward scheme, i.e.,

$$A^{*}=C^{(\nu)}-\beta D^{(\nu)}+\gamma\nabla_x G^{(\nu)}, \tag{17}$$

$$B^{*}=D^{(\nu)}+\beta C^{(\nu)}+\gamma\nabla_y G^{(\nu)}, \tag{18}$$

$$\phi^{*}=G^{(\nu)}+\gamma H(\nabla_x C^{(\nu)}+\nabla_y D^{(\nu)}), \tag{19}$$

$$A^{(\nu+1)}=C^{(\nu)}-\beta B^{*}+\gamma\nabla_x\phi^{*}, \tag{20}$$

$$B^{(\nu+1)}=D^{(\nu)}+\beta A^{*}+\gamma\nabla_y\phi^{*}, \tag{21}$$

and

$$\phi^{(\nu+1)}=\phi=G^{(\nu)}+\gamma H(\nabla_x A^{*}+\nabla_y B^{*}). \tag{22}$$

During a single cycle of the forward and backward forecasts, A and B are unknown variables, and the time-differenced formulas give rise to two equations in A and B. The final solution, for the case where sinusoidal spatial variation of quantities is assumed, turns out to be

$$A=-\gamma l/\beta)\phi,\ B=\gamma k/\beta)\phi. \tag{23}$$

Here, $k=2i\sin(m\Delta s)$ and $l=2i\sin(n\Delta s)$ where $m=2\pi/D$, and $n=2\pi/L$. The symbols L and D represent the wave lengths of harmonics in the x- and y-directions, respectively. This solution shows the geostrophic relation. The differences between the final solution, A and B, and the νth iterated solution, $A^{(\nu)}$ and $B^{(\nu)}$, are denoted $\Delta A^{(\nu)}$ and $\Delta B^{(\nu)}$, i.e.,

$$\Delta A^{(\nu)}=A-A^{(\nu)} \tag{24}$$

and

$$\Delta B^{(\nu)}=B-B^{(\nu)}. \tag{25}$$

Defining X as a convergent rate, we can write

$$\Delta A^{(\nu)}=X^{\nu}. \tag{26}$$

From equations (11) through (22), the quadratic equation for X is obtained. Two solutions for X indicate that the present scheme is conditionally convergent. The criterion for the convergence is

$$\Delta t<1/\sqrt{f^2+\hat{H}} \tag{27}$$

where $\hat{H}=H[\sin^2(m\Delta s)+\sin^2(n\Delta s)]/\Delta s^2$.

For a sinusoidal wave with a unit amplitude,

$$u=\sin m(x-ct) \tag{28}$$

where $m=2\pi/L$ and c is a phase speed, the Euler-backward scheme equation (2) turns out to be

$$u^{\tau+1}=Ru^{\tau} \tag{29}$$

where R is an amplification rate (Kurihara, 1965). For equations (4) and (5),

$$R=1-\sqrt{-1}\ (kc\Delta t)-(kc\Delta t)^2. \tag{30}$$

If we denote an angular frequency $\omega=kc$ and $i=\sqrt{-1}$, the amplification factor R_F for the forward forecast becomes

$$R_F=1-i\omega\Delta t-\omega^2\Delta t^2. \tag{31}$$

For the backward forecast,

$$R_B=1+i\omega\Delta t-\omega^2\Delta t^2. \tag{32}$$

An operation of a single cycle of the forward and backward forecast results in the amplification rate R_{FB} as follows (see fig. 2):

$$R_{FB}=R_F R_B=(1+i\omega\Delta t-\omega^2\Delta t^2)\ (1-i\omega\Delta t-\omega^2\Delta t^2)$$
$$=1-\omega^2\Delta t^2+\omega^4\Delta t^4. \quad (33)$$

If we adopt the modified Euler-backward iteration (Kurihara, 1965; Matsuno, 1966*b*), R_{FB} becomes

$$R_{FB}=1-\omega^2\Delta t^2+\tfrac{1}{4}\omega^6\Delta t^6. \quad (34)$$

If we choose a proper value of Δt that satisfies the computational stability condition, waves of $\omega\Delta t<1$ are reduced in amplitude. The damping is selectively effective for ω's of large value, i.e., for the high-frequency modes. The rate of damping is higher for the modified Euler-backward scheme than that for the original Euler-backward iteration.

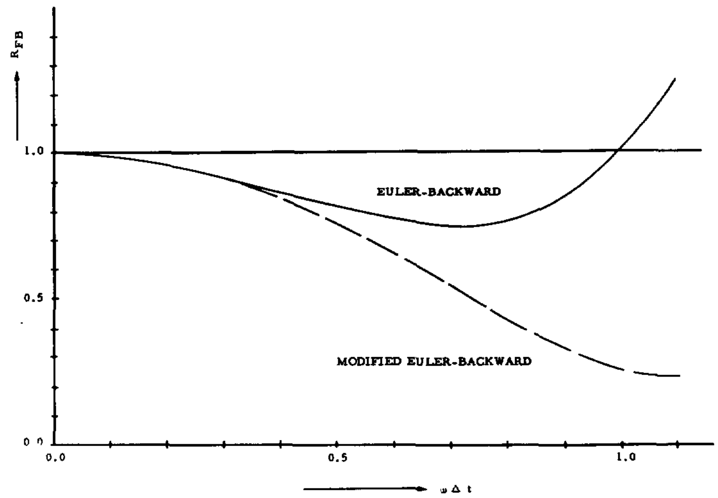


FIGURE 2.—Amplification rate of Euler-backward method with respect to $\omega\Delta t$.

4. INITIALIZATION OF A NONLINEAR SYSTEM

A numerical experiment of the proposed scheme was performed with a two-level primitive equation model (see Appendix). The model offered a fine opportunity to perform control experiments with a minimum of contaminating influences. Idealized atmospheric data were constructed from an extended forecast, and modifications were introduced at any earlier day to simulate the circumstances of an initialization. Problems involving observational error and inconsistencies between model and atmosphere were averted by this generation of fictitious data.

In the case presented here, geopotentials from the 17th day of the model atmosphere were used with the corresponding rotational wind components as first estimates for the initialization scheme. The process outlined above was carried out for almost 150 cycles. The resultant wind divergence was compared to the actual divergence field originally developed by the model. The two solutions (original and initialized) may be compared in figures 3 and 4.

We see that, from an initial guess of no divergence, the large-scale divergence field is recovered rather accurately. The close agreement of the zero line in the figures is most remarkable. The most evident fault of the method seems to be an inability to reproduce all of the amplitude of the irrotational components. Although the patterns are not shown, the situation is essentially the same in the lower layer of the model.

The corresponding vorticity fields in the upper layer are shown in figures 5 and 6. We note only that the rotational fields, as we can expect, are slightly damped during the iterations of forward and backward forecasts.

The inability of the method to recover the exact solution is indicated in figure 7 showing the root-mean-square error in the wind with respect to the iterations performed. This shows that, even with the close approximation suggested in the divergence and vorticity fields, somewhat less than half the error of the nondivergent wind was removed through the iteration technique.

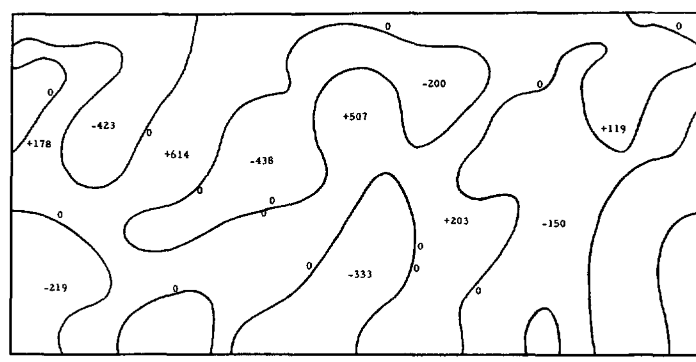


FIGURE 3.—Initialized divergence in the upper layer of the two-layer experimental model.

Although the process is still converging at 150 iterations, the rate is discouragingly slow.

The question remains unanswered whether or not the technique in its present design is accurate enough to be of practical use. One need may be a modified computational technique to speed convergence. Perhaps, further diagnosis of the features involved in the nonconverged portion of the solution (such as large-scale or small-scale features and rotational or irrotational components) will lend some insight in this direction.

5. FEASIBILITY OF APPLICATION OF INITIALIZATION METHODS TO OBJECTIVE ANALYSIS

The attainment of a balanced state between mass and velocity fields is a basic prerequisite of initial data for forecasts with the primitive equations, but in addition it is one of the basic goals in objective analysis. At present the simplest balance formulation, that based on the geostrophic relation, is widely used in objective analysis. Sasaki (1958) and Washington (1964) have suggested sophisticated generalizations based on the balance equation proposed by Charney (1955). Bedient, McDonell, and Shimomura (1967) are performing experiments also aimed at more generalized balanced analysis.

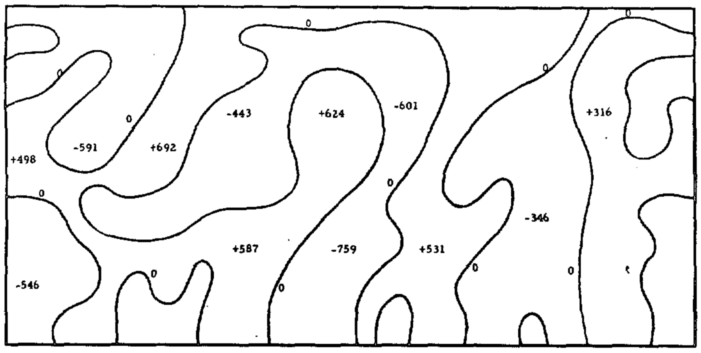


FIGURE 4.—Observed divergence in the upper layer of the two-layer experimental model.

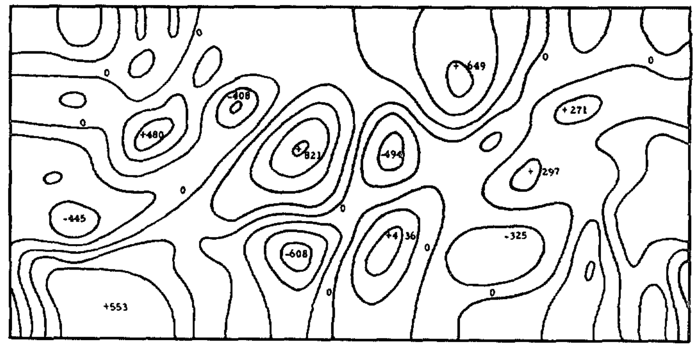


FIGURE 6.—Observed vorticity in the upper layer of the two-layer experimental model.

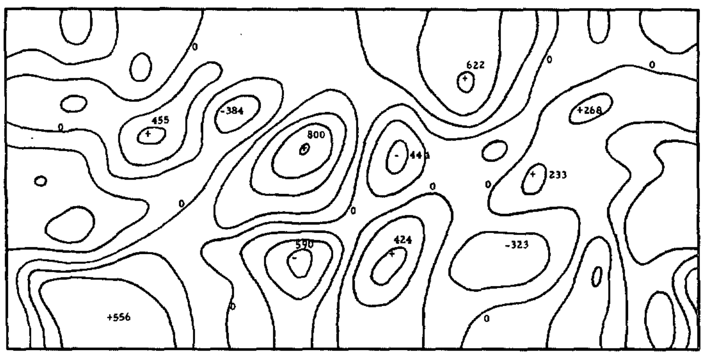


FIGURE 5.—Initialized vorticity in the upper layer of the two-layer experimental model.

The initialization method presented in the present paper may also have application in objective analysis and could be particularly effective if employed with a scheme to extrapolate information through time. For example, in operational forecasts, data could be carried along in time in the coordinate system of the forecast equations (e.g., on the σ-surface presently employed in most forecast models with the primitive equations). Then, observations could be introduced through conventional first-guess interpolation methods directly on the coordinate surfaces. Finally, the finer details of adjustment could be attained through use of an initialization approach of the type suggested here.

It seems to the authors that the approach outlined above has special advantages for the σ-type coordinate systems. It bypasses the need for an error-producing vertical interpolation of analyses from the more conventional p-system and combines analyses and initialization into one step. The latter characteristic, of course, is desirable in any coordinate system.

The generality of the approach mentioned above should make it useful in a wide range of physical problems. It requires no constraint on the divergence and thus should have a definite advantage over conventional methods in scales of motion where divergent modes are important. In particular, the very long waves of hemispheric scale should be handled more correctly; and, at the other end of the spectrum, mesoscale data initialization might be performed with more correct treatment of the gravity-inertia wave influence. Although an approximate balance of forces cannot be assumed in the mesoscale range, it seems possible that the proposed method could converge to the controlled state with acceleration terms consistent with the mesoscale (Yeh and Li, 1964).

Mutual adjustment may be simulated in addition to the idealized conditions of perfect wind-to-height or height-to-wind adjustment. Such options may be important in application to a global analysis initialization scheme where variation in physical modes and observational error may influence the relative importance of winds and heights. In middle latitudes where heights are more reliable, wind-to-height adjustment would be desirable. On the other hand, in the Tropics where wind observations become more meaningful, some procedure emphasizing the role of the wind field should be attempted, either through a height-to-wind process or a mutual adjustment technique. However, it is not a simple matter of data reliability that dictates the direction of adjustment. Fundamental physical principles are involved. For smaller scales (Rossby number well above unity), adjustment is dominated by the wind field. Scales where the Rossby number is below unity involve adjustment where heights tend to be dominating. Thus, the mere fact that winds in the Tropics are more reliable than the heights is no guarantee of successful height-to-wind adjustment, and this facet of the proposed initialization must be studied more thoroughly.

In the proposed scheme, one can visualize a more general analysis initialization which is both mathematically and physically consistent with the primitive forecast equations and able to account to some extent for variations in relative data accuracy as well as the many physical modes encountered on different scales.

APPENDIX

The two-layer primitive equation model used to manufacture data for the initialization study is similar to the model developed by Mintz (1964) at the University of California, Los Angeles. The present model departs in the following features:

1) A smaller area is used as the forecast domain.
2) No energy sources or sinks are included.
3) No orographic effect is included.
4) Potential temperature is carried as a history variable instead of temperature.

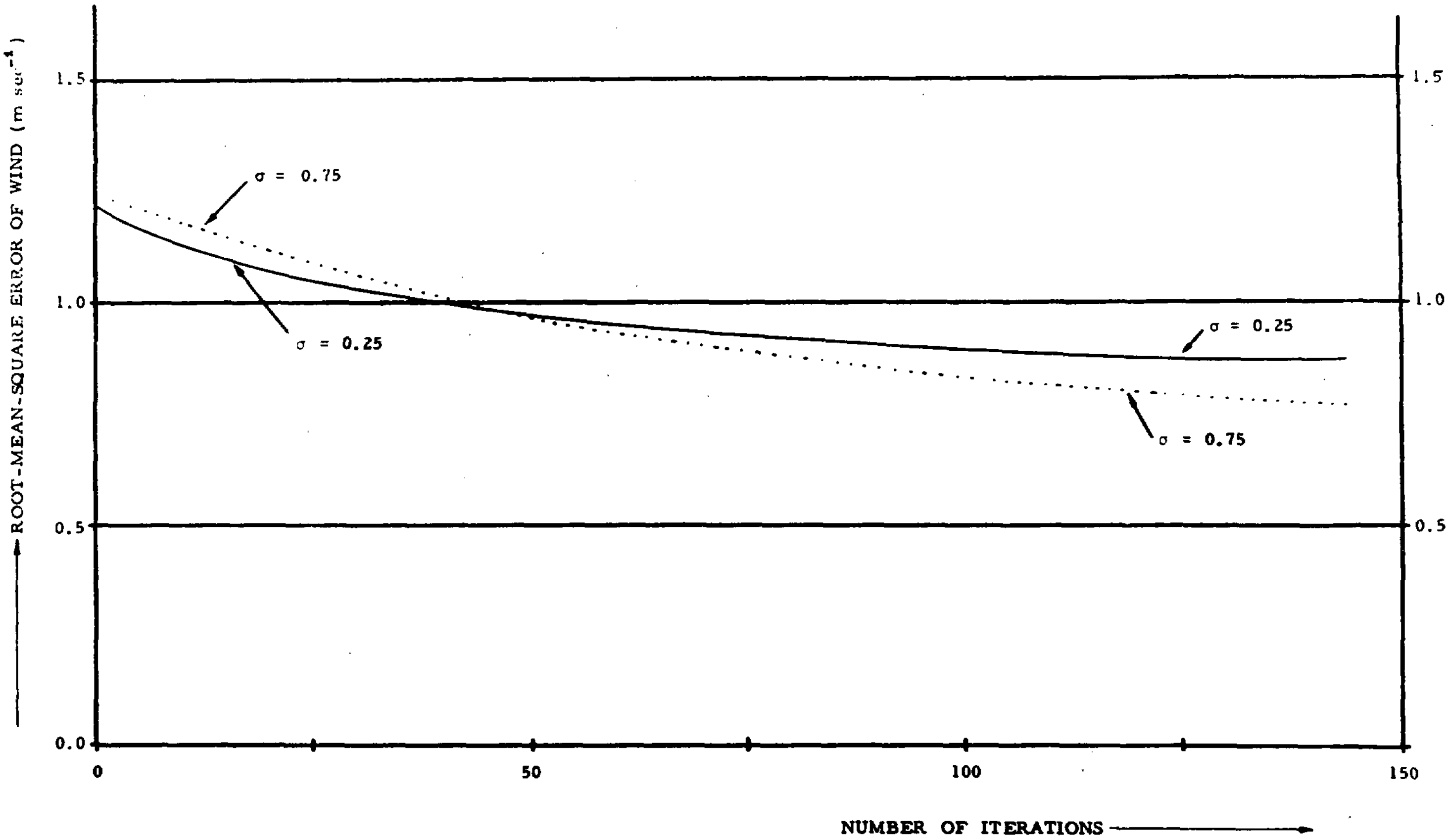


FIGURE 7.—Root-mean-square wind error as a function of iteration number.

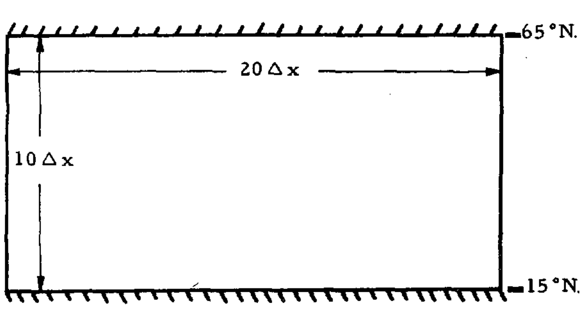


FIGURE 8.—Horizontal grid domain of two-layer model.

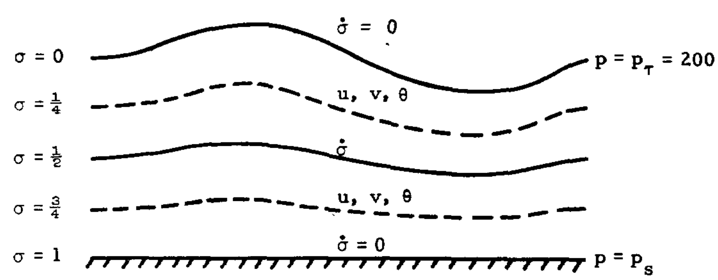


FIGURE 9.—Vertical grid domain of two-layer model.

5) A slightly different, finite space differencing in the continuity equation is used.

6) The dissipational effect is considered through skin friction and lateral diffusion in the free atmosphere. However, in some cases, the diffusion coefficients are reduced in magnitude.

The domain of integration is resolved with 20 by 10 grid net with a grid interval of 600 km. A 15-min time step is employed. A rigid, slippery wall condition is imposed at the north-south boundaries of the domain, at 65° N. and 15° N., respectively, on a Mercator projection (see fig. 8). Cyclic conditions are assumed at the east-west boundaries.

The vertical dimension is resolved in two layers in a σ-coordinate system. Placement of variables in the vertical may be seen in figure 9. The vertical σ-velocity, $\dot{\sigma}$, is set as zero at the surface (p_s= surface pressure) and at 200 mb ($p=p_T$).

Data for the reference atmosphere was created by a numerical integration of the model out to 26 days. Initial data for the integration consisted of a single long wave, 12000 km in length (at the latitude where the map factor equals 1), embedded in a zonal current with a baroclinically unstable vertical wind shear. During the forecast, the axis of the initial long wave begins to take on an upstream tilt with height. After about 10 days, waves of higher number began to predominate. At around 15 days, the reference atmosphere took on the appearance of a typical weather map. Additional baroclinic developments occurred as the calculation was extended to 26 days. The "observed" data for the initialization experiment were extracted from the reference atmosphere during the period of realistic appearance.

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